

Noncommutativity and Non-anticommutativity in Perturbative Quantum Gravity

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Abstract

In this paper we will study perturbative quantum gravity on supermanifolds with both noncommutative and non-anticommutative coordinates. We shall first analyse the BRST and the anti-BRST symmetries of this theory. Then we will also analyze the effect of shifting all the fields of this theory in background field method. We will construct a Lagrangian density which apart from being invariant under the extended BRST transformations is also invariant under on-shell extended anti-BRST transformations. This will be done by using the Batalin-Vilkovisky (BV) formalism. Finally, we will show that the sum of the gauge-fixing term and the ghost term for this theory can be elegantly written down in superspace with two Grassmann parameters.

Key words : BRST, anti-BRST, Perturbative Quantum Gravity
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1 Introduction

Noncommutative gauge theory first appeared due to NS tensor backgrounds [1]-[3]. Thus, many models based on the noncommutative gauge theories, including noncommutative standard model has been studied [4]-[5]. However, due to AdS/CFT correspondence a superconformal noncommutative gauge theory on the boundary of anti-de Sitter spacetime is dual to the noncommutative gravity in the bulk of that anti-de Sitter spacetime [6]-[9]. The RR background in string theory leads to a non-anticommutative gauge theory [10]-[13]. Thus, many models based on non-anticommutative gauge theory have also been studied [14]-[15]. Similarly, in analogy with noncommutative case, a superconformal non-anticommutative gauge theory on the boundary of anti-de Sitter spacetime will be dual to non-anticommutative gravity in the bulk of that anti-de Sitter spacetime. Hence, it will be interesting to study both commutative and non-anticommutative gravity in anti-de Sitter spacetime. Non-anticommutative gravity has already been studied in flat spacetime [16]-[18]. This work can provide the framework for analysing non-anticommutative gravity in anti-de Sitter spacetime. In fact, it is possible to analyse gravity with both noncommutative and non-anticommutative coordinates in the supermanifold formalism. Quantum field theory on spacetime with both noncommutative and

non-anticommutative coordinates has already been analyzed in the supermanifold formalism [19]. Unlike, noncommutative quantum field theory, this model of quantum field theory differs from the conventional quantum field theory even in absence of interactions. This is because even the Feynman propagator get modified due to the non-anticommutativity of the spacetime.

It will be interesting to generalise this work to anti-de Sitter spacetime and use it to study the model of gravity that is dual to both the gauge theories on NS and RR backgrounds. However, we will not do that in this paper. In this paper we will rather study some more formal aspects relating to gauge fixing of the noncommutative and non-anticommutative gravity in flat spacetime. This work along with the previous work done in this direction can provide the bases for analysing noncommutative and non-anticommutative gravity in anti-de Sitter spacetime which intern can be used to study an interesting example of AdS/CFT correspondence.

It may be remarked that non-anticommutativity in gravity has also occurs in complex spacetime [20]-[21]. These models of gravity on a complex spacetime were initially studied as attempts to unify gravity with electromagnetism [22]. However, they are now studied due to their relevance in string theory [23]-[25]. Apart from this many models of quantum gravity have suggested that the Planck's length might act like a minimum length scale for spacetime and this has led to the development of a modification of the General relativity called the Rainbow gravity [26]-[29]. Such a minimum length occurs naturally in a spacetime with a combination of noncommutative and non-anticommutative coordinates [18]. Another advantage of using noncommutative or non-anticommutative coordinates is that it gives spacetime a fuzzy structure [30]-[33]. This fuzziness of the spacetime can be used to solve the problems related to the occurrence of the singularity in a black hole [34]-[37].

All the degrees of freedom of perturbative quantum gravity on supermanifolds are not physical. Thus, before quantizing this theory we have to fix a gauge. This can be done at by adding a gauge fixing term and a ghost term to the original classical Lagrangian density. The effective Lagrangian density obtained by the sum of the gauge fixing term and the ghost term with the original classical Lagrangian density is invariant under BRST transformations. It is well known that any theory which is invariant under BRST transformations is also invariant under anti-BRST transformations. In these anti-BRST transformations, the role of ghosts and anti-ghosts is almost reversed.

Furthermore, in background field method all the field of the theory are shifted. The invariance of the theory under the original BRST and the original anti-BRST transformations along with these shift transformations can be analyzed in the Batalin-Vilkovisky (BV) formalism [38]-[41]. The BRST symmetry and shift symmetry for perturbative quantum gravity on non-anticommutative spacetime has been analyzed in superspace Batalin-Vilkovisky (BV) formalism [21]. Batalin-Vilkovisky (BV) formalism in the context of both the extended BRST, and the extended anti-BRST symmetries for Faddeev-Popov ghosts [43]-[45] along a superspace formalism for it is also well understood [46]-[48]. In this paper we will generalize the results that were obtained for the extended BRST invariance of the perturbative quantum gravity to include invariance under extended anti-BRST symmetry also. We shall also include noncommutativity of the spacetime coordinates, apart from the previously analyzed non-anticommutativity.

2 Deformed Supermanifolds

In this section we will analyze noncommutative and non-anticommutative supermanifolds. An elegant way to analyze perturbative quantum gravity with both noncommutative and non-anticommutative coordinates is in the language of supermanifolds [49]-[50]. The coordinates of the supermanifolds can be written as

$$z^a = x^a + y^a, \quad (1)$$

where x^a are the bosonic coordinates with even Grassmann parity,

$$[x^a, x^b] = 0, \quad (2)$$

and y^a are the fermionic coordinates with odd Grassmann parity,

$$\{y^a, y^b\} \equiv y^a y^b + y^b y^a = 0. \quad (3)$$

So the metric can be now written as

$$ds^2 = g_{ab} dz^a dz^b. \quad (4)$$

We want to impose noncommutativity and non-anticommutativity in such a way that the theory reduces to the usual noncommutative theory when there is no non-anticommutativity and it also reduces the usual non-anticommutative theory when there is no noncommutativity. This can be done by imposing the following relations

$$\begin{aligned} [\hat{z}^a, \hat{z}^b] &= 2y^a y^b + i\theta^{ab} + O(\theta^2), \\ \{\hat{z}^a, \hat{z}^b\} &= 2x^a x^b + 2i(x^a y^b + x^b y^a) - \tau^{ab} + O(\tau^2). \end{aligned} \quad (5)$$

Now in the limit $y^a \rightarrow 0$ and $\tau^{ab} \rightarrow 0$, we get

$$\begin{aligned} [\hat{x}^a, \hat{x}^b] &= i\theta^{ab}, \\ \{\hat{x}^a, \hat{x}^b\} &= 2x^a x^b \end{aligned} \quad (6)$$

and in the limit $x^a \rightarrow 0$ and $\theta^{ab} \rightarrow 0$, we get

$$\begin{aligned} [\hat{y}^a, \hat{y}^b] &= 2y^a y^b, \\ \{\hat{y}^a, \hat{y}^b\} &= \tau^{ab}. \end{aligned} \quad (7)$$

We use Weyl ordering and express the Fourier transformation of this metric as,

$$\hat{g}_{ab}^{(f)}(\hat{z}) = \int d^4 k \pi e^{-ik\hat{z}} g_{ab}^{(f)}(k). \quad (8)$$

Now we have a one to one map between a function of \hat{z} to a function of ordinary coordinates z via

$$g_{ab}^{(f)}(z) = \int d^4 k \pi e^{-ikx} g_{ab}^{(f)}(k). \quad (9)$$

$$g^{(f)ab}(z) \circ g_{ab}^{(f)}(z) = \exp(\omega^{ab} \partial_a^2 \partial_b^1) g^{(f)ab}(z_1) g_{ab}^{(f)}(z_2) \big|_{z_1=z_2=z}. \quad (10)$$

where ω^{ab} is a nonsymmetric tensor

$$\omega^{ab} = \tau^{ab} + \theta^{ab}. \quad (11)$$

Now $R_{bcd}^{(f)a}$ given as,

$$R_{bcd}^{(f)a} = -\partial_d \Gamma_{bc}^a + \partial_c \Gamma_{bd}^a + \Gamma_{ec}^a \circ \Gamma_{bd}^e - \Gamma_{ed}^a \circ \Gamma_{bc}^e, \quad (12)$$

and we also get $R_{bc} = R_{bcd}^d$. Thus, finally $R^{(f)}$ is given by

$$R^{(f)} = g^{(f)ab} \circ R_{ab}^{(f)}. \quad (13)$$

The Lagrangian density for pure gravity on the supermanifolds with cosmological constant λ can now be written as,

$$\mathcal{L}_c = \sqrt{g}^{(f)} \circ (R^{(f)} - 2\lambda), \quad (14)$$

where we have adopted units, such that $16\pi G = 1$.

3 Perturbative Quantum Gravity in BV Formulation

In perturbative gravity on flat spacetime one splits the full metric $g_{ab}^{(f)}(z)$ into $\eta_{ab}(z)$ which is the metric for the background flat spacetime and $h_{ab}(z)$ which is a small perturbation around the background spacetime.

$$g_{ab}^{(f)}(z) = \eta_{ab}(z) + h_{ab}(z). \quad (15)$$

Here both η_{ab} and h_{ab} are defined on a supermanifold. The covariant derivatives along with the lowering and raising of indices are compatible with the metric for the background spacetime. The small perturbation h_{ab} , is viewed as the field that is to be quantized.

All the degrees of freedom in h_{ab} are not physical as the Lagrangian density for h_{ab} is invariant under the following gauge transformations,

$$\begin{aligned} \delta_\Lambda h_{ab} &= D_{ab}^e \circ \Lambda_e \\ &= [\delta_b^e \partial_a + \delta_a^e \partial_b + g^{ce} \circ (\partial_c h_{ab}) + \\ &\quad g^{ec} \circ h_{ac} \partial_b + \eta^{ec} \circ h_{cb} \partial_a] \circ \Lambda_e. \end{aligned} \quad (16)$$

In order to remove these unphysical degrees of freedom, we need to fix a gauge by adding a gauge-fixing term along with a ghost term. In Landau gauge the sum of the gauge-fixing term and the ghost term can be expressed as [51]

$$\mathcal{L}_g = -\frac{i}{2} s \bar{s} (h^{ab} \circ h_{ab}) \quad (17)$$

Now the sum of the ghost term, the gauge fixing term and the original classical Lagrangian density is invariant under the following the BRST transformations

$$\begin{aligned} s h_{ab} &= D_{ab}^e \circ c_e, & s c^a &= c_b \circ \partial^b c^a, \\ s \bar{c}^a &= -b^a, & s b^a &= 0, \end{aligned} \quad (18)$$

and the following anti-BRST transformations

$$\begin{aligned} \bar{s} h_{ab} &= D_{ab}^e \circ c_e, & \bar{s} c^a &= b^a - c^b \circ \partial_b \bar{c}^a, \\ \bar{s} \bar{c}^a &= -\bar{c}^b \circ \partial_b \bar{c}^a, & \bar{s} b^a &= b^b \circ \partial_b \bar{c}^a, \end{aligned} \quad (19)$$

where

$$D_{ab}^e = \delta_b^e \partial_a + \delta_a^e \partial_b + g^{ce} \circ (\partial_c h_{ab}) + g^{ec} \circ h_{ac} \partial_b + \eta^{ec} \circ h_{cb} \partial_a. \quad (20)$$

BV-formalism is used to analyze the extended BRST and anti-BRST symmetries which is obtained by first shifting all the original fields as follows,

$$\begin{aligned} h_{ab} &\rightarrow h_{ab} - \tilde{h}_{ab}, \\ c^a &\rightarrow c^a - \tilde{c}^a, \\ \bar{c}^a &\rightarrow \bar{c}^a - \tilde{\bar{c}}^a, \\ b^a &\rightarrow b^a - \tilde{b}^a, \end{aligned} \quad (21)$$

and then requiring the resultant theory to be invariant under the original BRST and anti-BRST transformations along with these shift transformations. This can be achieved by letting the original fields to transform under the BRST transformations as

$$\begin{aligned} s h_{ab} &= \psi_{ab}, & s c^a &= \phi^a, \\ s \bar{c}^a &= \bar{\phi}^a, & s b^a &= \rho^a, \end{aligned} \quad (22)$$

and the shifted fields to transform under BRST transformations as

$$\begin{aligned} s \tilde{h}_{ab} &= \psi_{ab} - D_{ab}^e \circ c'_e, & s \tilde{c}^a &= \phi^a + c'_b \circ \partial^b c'_a, \\ s \tilde{\bar{c}}^a &= \bar{\phi}^a + b^{a'}, & s \tilde{b}^a &= \rho^a, \end{aligned} \quad (23)$$

where

$$\begin{aligned} h'_{ab} &= h_{ab} - \tilde{h}_{ab}, & c'^a &= c^a - \tilde{c}^a, \\ \bar{c}'^a &= \bar{c}^a - \tilde{\bar{c}}^a, & b'^a &= b^a - \tilde{b}^a. \end{aligned} \quad (24)$$

Here $\psi_{ab}, \phi^a, \bar{\phi}^a, \rho_a$ are ghosts associated with the shift symmetry. Their BRST transformations vanishes,

$$\begin{aligned} s \psi_{ab} &= 0, & s \phi^a &= 0 \\ s \bar{\phi}^a &= 0, & s \rho^a &= 0. \end{aligned} \quad (25)$$

We define antifields, with opposite parity corresponding to all the original fields. These antifields have following BRST transformations,

$$\begin{aligned} s h_{ab}^* &= n_{ab}, & s c^{*a} &= m^a, \\ s \bar{c}^{*a} &= \bar{m}^a, & s b^{*a} &= \bar{n}^a. \end{aligned} \quad (26)$$

The BRST transformations of these Nakanishi-Lautrup fields vanishes,

$$\begin{aligned} s n_{ab} &= 0, & s m^a &= 0, \\ s \bar{m}^a &= 0, & s \bar{n}^a &= 0. \end{aligned} \quad (27)$$

We also let the original fields to transform under anti-BRST transformations as

$$\begin{aligned} \bar{s} h_{ab} &= h_{ab}^* + D_{ab}^e \circ c'_e, \\ \bar{s} c^a &= c^{*a} + b^{a'} - 2c^{b'} \circ \partial_b \bar{c}^{a'}, \\ \bar{s} \bar{c}^a &= \bar{c}^{*a} - \bar{c}^{b'} \circ \partial_b \bar{c}^{a'}, \\ \bar{s} b^a &= b^{*a} + 2b^{b'} \circ \partial_b \bar{c}^{a'}, \end{aligned} \quad (28)$$

and the shifted fields to transform under the anti-BRST transformations as

$$\begin{aligned}\bar{s}\tilde{h}_{ab} &= h_{ab}^*, & \bar{s}\tilde{c}^a &= c^{*a}, \\ \bar{s}\tilde{\bar{c}}^a &= \bar{c}^{*a}, & \bar{s}\tilde{b}^a &= b^{*a}.\end{aligned}\quad (29)$$

The anti-BRST transformations of the ghost fields is given by

$$\begin{aligned}\bar{s}\psi_{ab} &= n_{ab} + 2D_{ab}^{e'} \circ b'_e - 2D_{ab}^{e'} \circ c^{b'} \circ \partial_b \bar{c}^{e'}, \\ \bar{s}\phi^a &= m^a - 2b^{b'} \circ \partial_b c^{a'} + 2\bar{c}^{b'} \circ \partial_b c^{c'} \circ \partial_c c^{a'}, \\ \bar{s}\bar{\phi}^a &= \bar{n}^a - 2b^{b'} \circ \partial_b \bar{c}^{a'}, \\ \bar{s}\tilde{\rho}^a &= m^{a'}.\end{aligned}\quad (30)$$

The anti-BRST transformations of the antifields and the Nakanishi-Lautrup fields vanishes

$$\begin{aligned}\bar{s}n_{ab} &= 0, & \bar{s}m^a &= 0, \\ \bar{s}\bar{m}^a &= 0, & \bar{s}\bar{n}^a &= 0.\end{aligned}\quad (31)$$

Furthermore, the anti-BRST transformation of the Nakanishi-Lautrup fields also vanishes

$$\begin{aligned}\bar{s}h_{ab}^* &= 0, & \bar{s}c^{*a} &= 0, \\ \bar{s}\bar{c}^{*a} &= 0, & \bar{s}b^{*a} &= 0.\end{aligned}\quad (32)$$

4 Extended Superspace Formulation

In this section we will express the sum of the gauge fixing term and the ghost term in superspace formalism by using two anti-commutating variable namely θ and $\bar{\theta}$. Now we can define the following superfields,

$$\begin{aligned}\omega_{ab} &= h_{ab} + \theta\psi_{ab} + \bar{\theta}(h_{ab}^* + D_{ab}^{e'} \circ \bar{c}'_e) \\ &\quad + \theta\bar{\theta}(n_{ab} + D_{ab}^{e'} \circ b_e - D_{ab}^{e'} \circ c^{e'} \circ \partial_c \bar{c}'_e), \\ \tilde{\omega}_{ab} &= \tilde{h}_{ab} + \theta(\psi_{ab} - D_{ab}^{e'} \circ c'_e) + \bar{\theta}h_{ab}^* + \theta\bar{\theta}n_{ab}, \\ \eta_a &= c_a + \theta\phi_a + \bar{\theta}(c_a^* + b_a' - 2b^{b'} \circ \partial_b \bar{c}_a') \\ &\quad + \theta\bar{\theta}(m_a - 2b^{b'} \circ \partial_b c_a' + c^{b'} \circ \partial_b \bar{c}^{c'} \circ \partial_c \bar{c}_a'), \\ \tilde{\eta}_a &= \tilde{c}_a + \theta(\phi_a + c_b' \partial^b c_a) + \bar{\theta}c_a^* + \theta\bar{\theta}m_a, \\ \bar{\eta}_a &= c_a + \theta\bar{\phi}_a + \bar{\theta}(\bar{c}_a^* - \bar{c}^{b'} \circ \partial_b \bar{c}_a') \\ &\quad + \theta\bar{\theta}(\bar{m}_a - 2b^{b'} \circ \partial_b \bar{c}_a'), \\ \tilde{\bar{\eta}}_a &= \tilde{\bar{c}}_a + \theta(\bar{\phi}_a + b_a') + \bar{\theta}\bar{c}_a^* + \theta\bar{\theta}\bar{m}_a,\end{aligned}\quad (33)$$

Thus we have

$$\begin{aligned}\frac{\partial^2}{\partial\theta\partial\theta}\tilde{\omega}^{*ab} \circ \tilde{\omega}_{ab} &= -n_{ab} \circ \tilde{h}^{ab} - h^{*ab}(\psi_{ab} - D_{ab}^{e'} \circ c'_e), \\ \frac{\partial^2}{\partial\theta\partial\theta}\tilde{\eta}_a^* \circ \tilde{\eta}^a &= -\bar{m}^a \tilde{c}_a + \bar{c}^{*a} \circ (\psi_a + c_b' \circ \partial^b c_a'),\end{aligned}$$

$$\begin{aligned}
-\frac{\partial^2}{\partial\bar{\theta}\partial\theta}\tilde{\eta}^a \circ \tilde{\eta}_a^* &= m^a\tilde{c}_a - c_a^* \circ (\bar{\phi}^a + b^{a'}), \\
-\frac{\partial^2}{\partial\bar{\theta}\partial\theta}\tilde{f}_a^* \circ \tilde{f}^a &= B'^a \circ \tilde{b}_a + b^{*a} \circ \rho_a.
\end{aligned} \tag{34}$$

Now we can express $\tilde{\mathcal{L}}_g$ as,

$$\tilde{\mathcal{L}}_g = \frac{\partial^2}{\partial\bar{\theta}\partial\theta}(\tilde{\omega}^{*ab} \circ \tilde{\omega}_{ab} + \tilde{\eta}_a^* \circ \tilde{\eta}^a - \tilde{\eta}^a \circ \tilde{\eta}_a^* - \tilde{f}_a^* \circ \tilde{f}^a). \tag{35}$$

Furthermore, we define Ψ as,

$$\Phi = \Psi + \theta s\Psi + \bar{\theta}\bar{s}\Psi + \theta\bar{\theta}s\bar{s}\Psi. \tag{36}$$

Thus, we can express \mathcal{L}_g as,

$$\mathcal{L}_g = \frac{\partial^2\Phi}{\partial\bar{\theta}\partial\theta}. \tag{37}$$

Now the complete Lagrangian in the superspace formalism is given by

$$\begin{aligned}
\mathcal{L} &= \frac{\partial^2\Phi}{\partial\bar{\theta}\partial\theta} + \frac{\partial^2}{\partial\bar{\theta}\partial\theta}(\tilde{\omega}_{ab}^* \circ \tilde{\omega}^{ab} + \tilde{\eta}_a^* \circ \tilde{\eta}^a - \tilde{\eta}^a \circ \tilde{\eta}_a^* - \tilde{\eta}^a \circ \tilde{\eta}_a^* - \tilde{f}_a^* \circ \tilde{f}^a) \\
&\quad + \mathcal{L}_c(h_{ab} - \tilde{h}_{ab}).
\end{aligned} \tag{38}$$

This Lagrangian density is manifestly invariant under the extended BRST transformations. Furthermore, it is also invariant under on-shell extended anti-BRST transformations.

5 Conclusion

In this paper we have analyzed perturbative quantum gravity with noncommutative and non-anticommutative coordinates on supermanifolds. After analyzing the BRST and the anti-BRST symmetries of this theory, we analyzed the extended BRST and the anti-BRST symmetries associated with it. Extended BRST and extended anti-BRST transformations were obtained by requiring the theory to be invariant under the original BRST and the original anti-BRST transformations along with the shift transformations. This was done by the using the BV-formulism. Then these extended BRST and anti-BRST symmetries were elegantly written down using extended superspace formulism.

The supermanifold formulism is ideally suited to study supersymmetric theories. So it will be interesting to perform a similar analyses on supergravity theories. It will also be interesting to generalize this work to anti-de Sitter spacetime as then we will be able to analyze the superconformal theories dual to this model of quantum gravity. The generalization of this work to anti-de Sitter spacetime can be easily done. This is because there are no infrared divergences in the ghosts in this spacetime. However, the generalization of this work to de Sitter spacetime will not be trivial, due to the infrared divergences in de Sitter spacetime [52]. However, if this problem can some how be resolved then we can possible analyze the implications of this model on inflation.

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